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FILM ABSORPTION ON A PLANE SURFACE IMBEDDED IN A GRANULATED MEDIUM

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1. Introduction. Absorption of a volatile component by an absorbent is one of the most important processes determining the efficiency of an absorption heat reformer (AHR). Thus according to [1] the main exergy losses (up to 45%) in lithium–bromide-operated absorption refrigerators (ALBR) are due to absorbers. Furthermore, insufficiency of heat and mass transfer in AHR absorbers leads to a considerable decrease in the cold productivity and hence a decrease in the total efficiency of the refrigerating pack as a whole.

It is notable that until recently the calculation of AHR absorber processes and their efficiency analysis have been performed without taking into account the fact that heat and mass transfer processes are going on jointly. This caused a considerable discrepancy between experimental and calculated values of the actual absorption processes. The coupled heat and mass transfer for absorption on a draining-down liquid film and on drops was first considered in [2, 3]. It was shown that the neglect of heat release during absorption in the lithium–bromide–water vapor system leads to a considerable overprediction of the mass transfer coefficient. At the same time, as follows from theoretical calculations, the smallness of the molecular transfer coefficients imposes some principal limitations on the methods of increasing the heat and mass transfer rates in film absorption processes.

A natural way of overcoming these limitations is to imbed the heat exchange surfaces in a granular bed. In this case the transfer coefficients increase considerably due to the dispersion terms. However, at the same time the hydraulic resistance to vapor seepage also experiences a drastic increase. Thus, mere qualitative speculations about the great potentials of such innovations are obviously insufficient, and no quantitative theories exist as yet. This has given impetus to the present research.

2. Physical and Mathematical Model of Heat and Mass Transfer in a Granular Bed. Let us consider nonisothermal absorption in a film on an inclined plane surface imbedded in a granular medium. The schematic of the flow and the coordinate system are shown in the Fig. 1.

The solution flows down the surface with volume rate of flow Q per unit width of the film. The constant relative mass concentration of the more volatile component C_0 and the temperature T_0 are given at the film inlet cross section, C_0 being smaller than the equilibrium mass concentration C_e at the given temperature T_0. Assuming that the conditions of thermodynamic equilibrium are satisfied on the surface of the film and using the Gibbs' phase rule we get the relation

$$C_s = \Omega(T_s, P), \tag{2.1}$$

where P is the medium pressure; the particular form of the function Ω is specified by the choice of the absorbent and of the substance which is absorbed. In the general case the dependence (2.1) is nonlinear. However, in the absorption processes the pressure remains constant and the temperature changes in a narrow range. Therefore, it is reasonable to keep only the linear part of the Taylor series expansion of (2.1):

$$C_s = \xi_1 + \xi_2 T_s, \tag{2.2}$$

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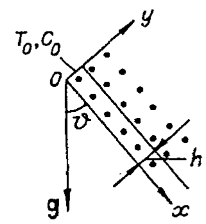

Fig. 1

where ξ_1, ξ_2 are the given functions of the pressure. As was noted above, the concentration C_0 at the initial section of the film is less than the equilibrium one C_e at the initial temperature T_0. Consequently, the inequality $C_0 < \xi_1(P) + \xi_2(P)T_0$ follows from (2.2).

Assuming that the film flow is in the steady state and that the granular medium is isotropic, let us write the equations of momentum, diffusion, and energy in the film as

$$\frac{\mu}{K}\,u + \frac{\rho c}{\sqrt{K}}\,u^2 = \rho g \cos\vartheta = \rho g_\vartheta; \tag{2.3}$$

$$u\,\frac{\partial C}{\partial x} = D_{\text{eff}}\,\frac{\partial^2 C}{\partial y^2}; \tag{2.4}$$

$$u\,\frac{\partial T}{\partial x} = a_{\text{eff}}\,\frac{\partial^2 T}{\partial y^2}, \tag{2.5}$$

where μ and ρ are the dynamical viscosity and the density of the solution, respectively, K and c are the permeability and the inertial coefficient, respectively, D_{eff} and a_{eff} are the effective diffusion and the thermal diffusivity coefficient, respectively. For close-packed beds of spherical particles with diameter d_p and porosity ε, the permeability is defined by the expression [4]

$$K = \frac{d_p^2\,\varepsilon^3}{150(1-\varepsilon)^2}.$$

We define the effective coefficients as follows [5]:

$$a_{\text{eff}} = a^0_{\text{eff}} + 0.1ud_p, \quad D_{\text{eff}} = 0.28D_L + 0.1ud_p, \quad \lambda^0_{\text{eff}} = \lambda_L\varepsilon + \lambda_p(1-\varepsilon),$$

where λ^0_{eff} is the effective thermal conductivity of granular medium in the absence of filtration, D_L is the coefficient of molecular diffusion, and λ_p is the thermal conductivity of the granular bed material.

Since the coefficients of Eq. (2.3) do not depend on the filtration rate, it can be solved explicitly with respect to u:

$$u = u_D\,\frac{\sqrt{1+4\mathrm{Ga}^*_\vartheta}-1}{2\mathrm{Ga}^*_\vartheta}, \tag{2.6}$$

where $u_D = Kg_\vartheta/\nu$ is the velocity determined by Darcy's law, $\Psi = (\sqrt{1+4\mathrm{Ga}^*_\vartheta}-1)/(2\mathrm{Ga}^*_\vartheta)$ is the multiplier, taking account of inertial effects, ν is the kinematic viscosity, and $\mathrm{Ga}^*_\vartheta = cK^{3/2}g_\vartheta/\nu^2$ is the modified Galileo number for the granular medium.

Values of the flow velocity, of the modified Galileo number, of the dispersion term, and of the granular bed's permeability are listed in Table 1 for some values of the granular diameter. According to experimental evidence [6] the inertial coefficient value $c = 0.55$ can be adopted for flows through different porous medium with Reynolds numbers $\mathrm{Re} = u\sqrt{K}/\nu$ up to 18.1. As the maximum value of Re, corresponding to the sixth mode of Table 1, is almost two times smaller ($\mathrm{Re} = 9.6$), the mentioned value $c = 0.55$ was adopted in all the subsequent calculations.

TABLE 1

Mode No.	d_p, mm	$K \cdot 10^9$, m^2	Ga*	u, $\frac{cm}{sec}$	u_D, $\frac{cm}{sec}$	Ψ	$0.1ud_p$, 10^{-7} $\frac{m^2}{sec}$
1	0.5	2.25	0.15	1.00	1.14	0.880	5.02
2	1.0	9.00	1.23	2.66	4.56	0.580	26.55
3	1.5	20.25	4.14	3.95	10.25	0.385	59.22
4	2.0	36.00	9.82	4.96	18.22	0.270	99.20
5	2.5	56.30	19.19	5.80	28.47	0.200	145.02
6	3.0	81.00	33.15	6.53	41.00	0.159	195.88

Let us dwell a bit more on the choice of the value of porosity ε. It is well known [5] that for the central part of the close-packed bed of spherical particles the value $\varepsilon = 0.4$ can be accepted. But, in a particular case, the flow proceeds in a narrow layer immediately adjacent to the wall surface, just where the structure of the granular packing is changed because of the wall presence. As a consequence the porosity ε increases considerably.

According to [5], in a granular bed composed of smooth balls the structure of only the layer immediately adjacent to the wall is distorted. For this layer the value $\varepsilon = 0.6$ may be adopted as a mean porosity value, which will be used in further calculations.

It follows from the filtration rate expression (2.6) that at $4\mathrm{Ga}_\vartheta^* \ll 1$ one can neglect in Eq. (2.3) the term quadratic in velocity. Let us estimate from the condition $4\mathrm{Ga}_\vartheta^* = 1$ the bed granular's diameter, beginning from which a significant deviation from Darcy's law is observed:

$$d_p = \left(\frac{\nu^2}{2.2\,\omega g_\vartheta}\right)^{1/3}, \qquad \omega = \left[\frac{\varepsilon^3}{150\,(1-\varepsilon)^2}\right]^{3/2}. \tag{2.7}$$

For the lithium bromide solution at $t = 35$°C with nonvolatile component concentration $\xi = 51.12\,\%$ ($\nu = 1.936 \cdot 10^{-6}$ m^2/sec) it follows from (2.7) that $d_p = 0.588$ mm (this value corresponds to the case of a film falling down along a vertical surface, that is, $\vartheta = 0$). Hence, in most cases of practical importance ($d_p \geqslant 1$ mm) the inertial effects must be taken into account, that is, Eq. (2.3) must be used.

Let us proceed now to the statement of the boundary value problem. As has been mentioned already the temperature T_0 and concentration C_0 are given at the inlet cross section ($x = 0$) of the film:

$$T(0,y) = T_0, \qquad C(0,y) = C_0. \tag{2.8}$$

Assuming that heat is transferred to the film only with the absorbing vapor mass, the following relation at the free film surface can be written:

$$\lambda_{\mathrm{eff}} \left.\frac{\partial T}{\partial y}\right|_{y=h} = \rho r_{\mathrm{a}} D_{\mathrm{eff}} \left.\frac{\partial C}{\partial y}\right|_{y=h}, \tag{2.9}$$

where r_{a} is the absorption heat; $h = Q/u$ is the thickness of the solution film determined by the given mass flow rate Q. For the diffusion problem an apparent impermeability condition must be imposed on the solid surface at $y = 0$:

$$\left.\frac{\partial C}{\partial y}\right|_{y=0} = 0, \tag{2.10}$$

while for the heat transfer problem two types of boundary conditions are of interest: isothermal and adiabatic surfaces.

For these cases the boundary conditions are, respectively,

$$T(x,0) = T_w; \tag{2.11}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0. \tag{2.12}$$

Let us nondimensionalize the problem (2.3)–(2.5),(2.8)–(2.12) by introducing the following dimensionless parameters and variables

$$\overline{y} = \frac{y}{h}, \quad \overline{x} = \frac{x}{\mathrm{Pe}_Q^* h}, \quad \overline{T} = \frac{T - T_0}{T_e - T_0}, \quad \overline{C} = \frac{C - C_0}{C_e - C_0}, \tag{2.13}$$

where $C_e = \xi_1 + \xi_2 T_0$; $C_0 = \xi_1 + \xi_2 T_e$; $\mathrm{Pe}_Q^* = Q/a_{\text{eff}}$. These result in

$$\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} = \frac{\partial \overline{T}}{\partial \overline{x}}, \qquad \frac{\partial^2 \overline{C}}{\partial \overline{y}^2} = \frac{1}{\mathrm{Le}^*}\frac{\partial \overline{C}}{\partial \overline{x}}; \tag{2.14}$$

$$\overline{T}(0,\overline{y}) = 0, \qquad \overline{C}(0,\overline{y}) = 0; \tag{2.15}$$

$$\overline{C}(\overline{x},1) = 1 - \overline{T}(\overline{x},1); \tag{2.16}$$

$$\left.\frac{\partial \overline{T}}{\partial \overline{y}}\right|_{\overline{y}=1} = \mathrm{Le}^* K_a \left.\frac{\partial \overline{C}}{\partial \overline{y}}\right|_{\overline{y}=1}; \tag{2.17}$$

$$\overline{T}(\overline{x},0) = \overline{T}_w; \tag{2.18}$$

$$\left.\frac{\partial \overline{T}}{\partial \overline{y}}\right|_{\overline{y}=0} = 0, \tag{2.19}$$

where $\mathrm{Le}^* = D_{\text{eff}}/a_{\text{eff}}$ is the modified Lewis number; $K_a = r_a(C_e - C_0)/c_p(T_e - T_0)$. The system (2.14)–(2.19) is a boundary value problem with the fourth type boundary conditions. Hence, the Laplace transform method is the most suitable one for its solution.

3. Isothermal Surface. Let us consider the case of an isothermal surface. Applying the Laplace transform in $\overline{x}$ variable to Eq. (2.14) and to boundary conditions (2.16)–(2.18) we obtain, respectively,

$$\frac{d^2 \tilde{\overline{C}}}{d\overline{y}^2} = \frac{1}{\mathrm{Le}^*} s \tilde{\overline{C}}, \qquad \frac{d^2 \tilde{\overline{T}}}{d\overline{y}^2} = s \tilde{\overline{T}}; \tag{3.1}$$

$$\tilde{\overline{T}}(\overline{s},0) = \frac{\overline{T}_w}{s}, \qquad \left.\frac{d\tilde{\overline{C}}}{d\overline{y}}\right|_{\overline{y}=0} = 0; \tag{3.2}$$

$$\tilde{\overline{C}}(\overline{s},1) = \frac{1}{s} - \tilde{\overline{T}}(\overline{s},1), \qquad \left.\frac{d\tilde{\overline{T}}}{d\overline{y}}\right|_{\overline{y}=1} = \mathrm{Le}^* K_a \left.\frac{d\tilde{\overline{C}}}{d\overline{y}}\right|_{\overline{y}=1} \tag{3.3}$$

The general solutions of Eq. (3.1) are of the form

$$\tilde{\tilde{C}}= M_1 \cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\,\overline{y} + M_2 \sinh\sqrt{\frac{s}{\mathrm{Le}^*}}\,\overline{y}, \qquad \tilde{\tilde{T}}= N_1 \cosh\sqrt{s}\,\overline{y} + N_2 \sinh\sqrt{s}\,\overline{y}.$$

By satisfying conditions (3.2), (3.3) we find the solution to the boundary value problem (3.1)–(3.3) in the image space:

$$\tilde{\tilde{C}}= \frac{1}{s}\,\frac{(\cosh\sqrt{s}-\overline{T}_w)\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\,\overline{y}}{\left(\sqrt{\mathrm{Le}^*}\,K_a \sinh\sqrt{s}\sinh\sqrt{\frac{s}{\mathrm{Le}^*}}+\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh\sqrt{s}\right)} = \frac{\Phi_1(s)}{s\varphi(s)}; \tag{3.4}$$

$$\tilde{\tilde{T}}= \frac{1}{s}\,\frac{\overline{T}_w\sqrt{\mathrm{Le}^*}\,K_a \sinh\sqrt{\frac{s}{\mathrm{Le}^*}}\sinh[\sqrt{s}(1-\overline{y})]+\overline{T}_w\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh[\sqrt{s}(1-\overline{y})]}{\left(\sqrt{\mathrm{Le}^*}\,K_a \sinh\sqrt{s}\sinh\sqrt{\frac{s}{\mathrm{Le}^*}}+\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh\sqrt{s}\right)}$$

$$+\frac{1}{s}\,\frac{\sqrt{\mathrm{Le}^*}\,K_a \sinh\sqrt{\frac{s}{\mathrm{Le}^*}}\sinh\sqrt{s}\,\overline{y}}{\left(\sqrt{\mathrm{Le}^*}\,K_a \sinh\sqrt{s}\sinh\sqrt{\frac{s}{\mathrm{Le}^*}}+\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh\sqrt{s}\right)} = \frac{\Phi_2(s)}{s\varphi(s)}. \tag{3.5}$$

We determine the temperature and concentration profiles at the initial section ($x \to 0$) and at infinity ($x \to \infty$) without performing an inverse transform to the original space by making use of the Laplace transform initial- and final-value theorems (Tauberian properties):

$$\lim_{s\to\infty} sF(s) \to f(0), \qquad \lim_{s\to 0} sF(s) \to f(\infty). \tag{3.6}$$

A distinct peculiarity of the problem in question is that there are three boundary layers in the initial section, namely, the heat and diffusion ones at the free surface and only the heat layer near the wall. Using the first of relations (3.6) we find an asymptote in the domain $D_0 = \{(x,y) \mid x \to 0,\ y \in O_\delta(0)\}$:

$$\overline{T}(D) \simeq \mathcal{L}^{-1}\left[\lim_{\substack{s\to\infty\\ y\in O_\delta(0)}} \frac{\Phi_2(s)}{\varphi(s)}\right]. \tag{3.7}$$

At large values of the parameter s let us set $\sinh s = \cosh s = e^s/2$. Since $y \in O_\delta(0)$, one can neglect the last term in Φ_2, whereby we get from (3.7)

$$\overline{T}(D_0) \simeq \mathcal{L}^{-1}\left(\frac{\overline{T}_w}{s}\,e^{-\overline{y}\sqrt{s}}\right) = \overline{T}_w \operatorname{erfc}\left(\frac{\overline{y}}{2\sqrt{\overline{x}}}\right).$$

In a similar way we find an asymptote for the domain $D_1 = \{(x,y) \mid x \to 0,\ y \in O_\delta(1)\}$:

$$\overline{T}(D_1) \simeq \mathcal{L}^{-1}\left[\lim_{\substack{s\to\infty\\ y\in O_\delta(1)}} \frac{\Phi_2(s)}{\varphi(s)}\right] = \frac{\sqrt{\mathrm{Le}^*}\,K_a}{\sqrt{\mathrm{Le}^*}\,K_a+1}\mathcal{L}^{-1}\left[\frac{1}{s}\,e^{-\sqrt{s}(1-\overline{y})}\right] = \frac{\sqrt{\mathrm{Le}^*}\,K_a}{\sqrt{\mathrm{Le}^*}\,K_a+1}\operatorname{erfc}\left[\frac{1-\overline{y}}{2\sqrt{\overline{x}}}\right]; \tag{3.8}$$

$$\overline{C}(D_1) \simeq \mathcal{L}^{-1}\left[\lim_{\substack{s\to\infty\\ y\in O_\delta(1)}} \frac{\Phi_1(s)}{\varphi(s)}\right] = \frac{1}{\sqrt{\mathrm{Le}^*}K_a+1}\mathcal{L}^{-1}\left[\frac{1}{s}\,e^{-\sqrt{s/\mathrm{Le}^*}(1-\overline{y})}\right] = \frac{1}{\sqrt{\mathrm{Le}^*}\,K_a+1}\operatorname{erfc}\left[\frac{1-\overline{y}}{2\sqrt{\mathrm{Le}^*\overline{x}}}\right]. \tag{3.9}$$

The asymptotical values of temperature and concentration at infinity can be easily obtained from (3.4), (3.5) and from the second equation of (3.6) by letting s tend to zero and keeping the linear terms of the hyperbolic functions expansions: $\cosh s \simeq 1$, $\sinh s \simeq s$. Hence,

$$\overline{C}_\infty = 1 - \overline{T}_w, \qquad \overline{T}_w = \overline{T}_\infty. \tag{3.10}$$

Making use of (2.13) one can easily check that the equilibrium equation (2.2) is satisfied by solution (3.10). Applying (3.8), (3.9) we find the heat and mass flows inward of the film depth:

$$j = -\rho D_{\text{eff}} \frac{\partial C}{\partial y}\bigg|_{y=h} = \frac{1}{\sqrt{\pi}} \frac{\rho D_{\text{eff}}(C_e - C_0)}{1 + \sqrt{\dfrac{D_{\text{eff}}}{a_{\text{eff}}} \dfrac{r_a(C_e - C_0)}{c_p(T_e - T_0)}}} \sqrt{\frac{u}{D_{\text{eff}}\, x}}; \tag{3.11}$$

$$q = -\lambda_{\text{eff}} \frac{\partial T}{\partial y}\bigg|_{y=h} = \frac{1}{\sqrt{\pi}} \frac{\lambda_{\text{eff}}(T_e - T_0)}{1 + \sqrt{\dfrac{a_{\text{eff}}}{D_{\text{eff}}} \dfrac{c_p(T_e - T_0)}{r_a(C_e - C_0)}}} \sqrt{\frac{u}{a_{\text{eff}}\, x}}. \tag{3.12}$$

Local values of the Nusselt and Sherwood numbers on the free film surface are obtained from (3.11), (3.12):

$$\text{Nu} = \frac{\alpha x}{\lambda_L} = \frac{qx}{\lambda_L(T_e - T_0)} = \frac{a_{\text{eff},L}}{\sqrt{\pi}} \frac{\sqrt{\text{Le}^*}\, K_a}{\sqrt{\text{Le}^*}\, K_a + 1} \sqrt{\text{Pe}_x^*},$$

$$\text{Sh} = \frac{\beta x}{D_L} = \frac{jx}{\rho D_L(C_e - C_0)} = \frac{1}{\sqrt{\pi}} \frac{D_{\text{eff},L}}{\sqrt{\text{Le}^*}(\sqrt{\text{Le}^*}\, K_a + 1)} \sqrt{\text{Pe}_x^*},$$

where $\text{Pe}_x^* = ux/a_{\text{eff}}$; α, β are the coefficients of heat and mass transfer, respectively. For the plate length-averaged values of the Nusselt and Sherwood numbers, we get

$$\overline{\text{Nu}} = \frac{1}{L}\int_0^L \text{Nu}\, dx = \frac{2a_{\text{eff},L}}{3\sqrt{\pi}} \frac{\sqrt{\text{Le}^*}\, K_a}{\sqrt{\text{Le}^*}\, K_a + 1} \sqrt{\text{Pe}_L^*}; \tag{3.13}$$

$$\overline{\text{Sh}} = \frac{1}{L}\int_0^L \text{Sh}\, dx = \frac{2D_{\text{eff},L}}{3\sqrt{\pi}} \frac{1}{\sqrt{\text{Le}^*}(\sqrt{\text{Le}^*}\, K_a + 1)} \sqrt{\text{Pe}_L^*} \qquad \left(\text{Pe}_L^* = \frac{uL}{a_{\text{eff}}}\right). \tag{3.14}$$

As follows from (3.13) and (3.14), the heat and mass transfer does not depend on the solution flow rate in the initial section of the flow.

Let compare the heat and mass transfer rate of the problem in question with that of the problem of nonisothermal absorption in the initial section of a film freely flowing down a smooth plate. For the latter case the averaged values of the Nusselt and Sherwood numbers have been obtained in [3] and can be written as

$$\overline{\text{Nu}^0} = \sqrt{\frac{6}{\pi}} \frac{\sqrt{\text{Le}}\, K_a}{\sqrt{\text{Le}}\, K_a + 1} \left(\frac{\text{Ra}_\vartheta}{3}\right)^{1/6} \text{Pe}_Q^{1/2}; \tag{3.15}$$

$$\overline{\text{Sh}^0} = \sqrt{\frac{6}{\pi}} \frac{1}{\sqrt{\text{Le}}(\sqrt{\text{Le}}\, K_a + 1)} \left(\frac{\text{Ra}_\vartheta}{3}\right)^{1/6} \text{Pe}_Q^{1/2}, \tag{3.16}$$

where $\text{Pe}_Q = Q/a_L$; Q is the volume rate of flow per film unit width; $\text{Ra}_\vartheta = g_\vartheta L^3/(\nu a_L)$ is the Rayleigh number.

Representing the Lewis number as $\text{Le}^* = \text{Le} D_{\text{eff},L}\, a_{L,\text{eff}}$ (where $f_{\alpha,\beta}$ denotes the ratio $f_{\alpha,\beta} = f_\alpha/f_\beta$) and using (3.13)–(3.16), we can write the ratio of the averaged Nusselt and Sherwood numbers for the granular medium case to those for flow over a smooth plate

$$\eta = \frac{\overline{\text{Nu}}}{\overline{\text{Nu}^0}} = \frac{\overline{\text{Sh}}}{\overline{\text{Sh}^0}} = 0.33\, \frac{\sqrt{\text{Le}}\, K_a + 1}{\sqrt{\text{Le}^*}\, K_a + 1} D_{\text{eff},L}^{1/2} \text{Da}^{1/2} \text{Ra}_\vartheta^{1/3} \Psi^{1/2} \text{Pe}_Q^{-1/2}. \tag{3.17}$$

Expressions (3.15) and (3.16) are obtained for the laminar film flow case. According to [7], the following Reynolds number range corresponds to this mode:

$$\text{Re} \leqslant 0.47(\text{Fi})^{1/10}$$

TABLE 2

T_0, K	T_e, K	C_0	C_e	$a_L \cdot 10^7$, m²/sec	c_p, $\frac{\text{J}}{\text{kg K}}$	r_a, $\frac{\text{kJ}}{\text{kg}}$	ρ_L, $\frac{\text{kg}}{\text{m}^3}$	$\nu_L \cdot 10^6$, m²/sec	$\mu \cdot 10^3$, Pa·sec	λ_L, $\frac{\text{W}}{\text{m K}}$	$D_L \cdot 10^9$, m²/sec	Pr	Le	$\sigma \cdot 10^2$, N/m	K_a
308	311.2	0.47	0.49	1.38	2128	2641	1544	1.936	3.0	0.455	2.355	14	0.017	8.52	7.3

($\mathrm{Fi} = \sigma^3/(g\nu^4\rho^3)$ is the Kapitsa number).

For a lithium bromide solution with temperature and concentration corresponding to the parameters of an ALBR absorber working substance (Table 2), we obtain from (3.17) that $\mathrm{Re} \leqslant 3.8$ and conseqently $\mathrm{Pe}_Q = \mathrm{Re}\,\mathrm{Pr} \leqslant 54$. Because the maximum values of $\overline{\mathrm{Nu}}^0$, $\overline{\mathrm{Sh}}^0$ are achieved at $\mathrm{Pe}_Q = 54$, the calculations of the relative intensity η were done only for this Peclet number value. The results are listed in Table 3.

From Tables 1 and 3 it is clear that at $d_p \geqslant 1$ mm, that is, when the flow mode deviates significantly from Darcy's law ($\Psi \simeq 0.58$), heat and mass transfer of the granular medium begin to prevail, and at $d_p = 3$ (mode 6) the relative intensity η reaches an almost fourfold value. Hence, application of a granular bed enables one to achieve significant intensification of heat and mass exchange processes at the initial section of the film. However, neither the length of the initial section of the film nor the heat and mass exchange at the main section of the film can be estimated from the obtained asymptotic solutions (3.8) and (3.9). To determine these quantities, it is necessary to perform the inverse Laplace transform of (3.4) and (3.5). As expressions (3.4) and (3.5) are the quotients of two generalized polynomials, the expansion theorem (Heaviside's partial fraction rule) can be applied. Except for the obvious zero root $s_0 = 0$, the denominator has a countable number of simple roots s_n, which are determined by the equation $\varphi(s) = 0$:

$$\sqrt{\mathrm{Le}^*}\, K_a \sinh\sqrt{s}\,\sinh\sqrt{\frac{s}{\mathrm{Le}^*}} + \cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\,\cosh\sqrt{s} = 0. \tag{3.18}$$

Making use of the relations

$$\cosh z = \cos iz, \qquad \sinh z = \frac{\sin iz}{i}, \tag{3.19}$$

we obtain from (3.18)

$$\cos i\sqrt{s}\,\cos i\sqrt{\frac{s}{\mathrm{Le}^*}} - \sqrt{\mathrm{Le}^*}\, K_a \sin i\sqrt{s}\,\sin i\sqrt{\frac{s}{\mathrm{Le}^*}} = 0. \tag{3.20}$$

Introducing the new variable $\mu = i\sqrt{s/\mathrm{Le}^*}$, we can find the characteristic equation from (3.20):

$$\cos\mu\cos\sqrt{\mathrm{Le}^*}\,\mu - \sqrt{\mathrm{Le}^*}\, K_a \sin\mu\sin\sqrt{\mathrm{Le}^*}\,\mu = 0,$$

which can be rewritten as

$$\tan\mu\tan\sqrt{\mathrm{Le}^*}\,\mu = \frac{1}{\sqrt{\mathrm{Le}^*}\, K_a}. \tag{3.21}$$

According to the expansion theorem, the solution in the original space is given by the expressions

$$\overline{C} = \sum_{n=0}^{\infty} \frac{\Phi_1(s_n)}{[s\varphi(s)]'_{s_n}}\, e^{s_n x}, \qquad \overline{T} = \sum_{n=0}^{\infty} \frac{\Phi_2(s_n)}{[s\varphi(s)]'_{s_n}}\, e^{s_n x}. \tag{3.22}$$

Let us calculate the denominator of (3.22) explicitly:

$$[s\varphi(s)]'_{s_n} = \left[\sqrt{\mathrm{Le}^*}\, K_a \sinh\sqrt{s}\,\sinh\sqrt{\frac{s}{\mathrm{Le}^*}} + \cosh\sqrt{s}\,\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\right]_{s_n} + \frac{1}{2}\sqrt{\mathrm{Le}^*}\, K_a \sqrt{s_n}\cosh\sqrt{s_n}\,\sinh\sqrt{\frac{s_n}{\mathrm{Le}^*}}$$
$$+\frac{1}{2}\sqrt{\mathrm{Le}^*}\, K_a\sqrt{\frac{s_n}{\mathrm{Le}^*}}\sinh\sqrt{s_n}\,\cosh\sqrt{\frac{s_n}{\mathrm{Le}^*}} + \frac{1}{2}\sqrt{s_n}\cosh\sqrt{s_n}\,\sinh\sqrt{\frac{s_n}{\mathrm{Le}^*}} + \frac{1}{2}\sqrt{\frac{s_n}{\mathrm{Le}^*}}\cosh\sqrt{s_n}\,\sinh\sqrt{\frac{s_n}{\mathrm{Le}^*}}, \tag{3.23}$$

TABLE 3

Mode No.	Le^*	d_p, mm	$\mathrm{Da}^{1/2}\mathrm{Ra}^{1/3}$	$D_{\mathrm{eff},L}$	$\eta = \frac{\overline{\mathrm{Nu}}}{\overline{\mathrm{Nu}}^0} = \frac{\overline{\mathrm{Sh}}}{\overline{\mathrm{Sh}}^0}$
1	0.75	0.5	1.57	213	0.26
2	0.94	1.0	3.14	1128	0.87
3	0.97	1.5	4.72	2515	1.57
4	0.98	2.0	6.30	4213	2.26
5	0.9885	2.5	7.87	6158	2.95
6	0.992	3.0	9.45	8318	3.63

where the bracketed expression on the right-hand side of (3.23) is equal to zero by virtue of (3.18). Taking into account the relations

$$\sinh\sqrt{s_n} = \frac{1}{i}\sin\sqrt{\mathrm{Le}^*}\,\mu_n, \qquad \sinh\sqrt{\frac{s_n}{\mathrm{Le}^*}} = \frac{1}{i}\sin\mu_n,$$

$$\cosh\sqrt{s_n} = \cos\sqrt{\mathrm{Le}^*}\,\mu_n, \qquad \cosh\sqrt{\frac{s_n}{\mathrm{Le}^*}} = \cos\mu_n,$$

which are consequences of the dependence $\mu_n = i\sqrt{s_n/\mathrm{Le}^*}$ and of (3.19), we obtain finally from (3.23)

$$\lim_{s\to s_n}[s\varphi(s)]' = -\frac{\mu_n}{2}\Big\{(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\,\mu_n\sin\mu_n + \sqrt{\mathrm{Le}^*}\,(K_a+1)\sin\sqrt{\mathrm{Le}^*}\,\mu_n\cos\mu_n\Big\}.$$

Thus the general solution to the problem with an isothermal surface is given by the following expressions for the concentration and temperature distributions inside the film:

$$\overline{C} = (1-\overline{T}_w) - 2\sum_{n=1}^{\infty}\frac{(\cos\sqrt{\mathrm{Le}^*}\mu_n - \overline{T}_w)\cos\mu_n\,\overline{y}\,e^{-\mathrm{Le}^*\mu_n^2\overline{x}}}{\mu_n\Big[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_n\sin\mu_n+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_n\cos\mu_n\Big]}; \tag{3.24}$$

$$\overline{T} = \overline{T}_w - 2\sum_{n=1}^{\infty}\Bigg\{\frac{\overline{T}_w\cos\mu_n\cos\sqrt{\mathrm{Le}^*}\mu_n(1-\overline{y})\,e^{-\mathrm{Le}^*\mu_n^2\overline{x}}}{\mu_n\Big[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_n\sin\mu_n+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_n\cos\mu_n\Big]}$$
$$-\frac{\overline{T}_w\sqrt{\mathrm{Le}^*}\,K_a\sin\mu_n\sin\sqrt{\mathrm{Le}^*}\mu_n(1-\overline{y})e^{-\mathrm{Le}^*\mu_n^2\overline{x}}}{\mu_n\Big[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_n\sin\mu_n+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_n\cos\mu_n\Big]}$$
$$-\frac{\sqrt{\mathrm{Le}^*}\,K_a\sin\mu_n\sin\sqrt{\mathrm{Le}^*}\,\mu_n\,\overline{y}e^{-\mathrm{Le}^*\mu_n^2\overline{x}}}{\mu_n\Big[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_n\sin\mu_n+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_n\cos\mu_n\Big]}\Bigg\}. \tag{3.25}$$

4. Analysis of the Solution. Since the sequence of the roots $\{\mu_i\}$ of the characteristic equation (3.21) is monotonically increasing, that is, the inequality

$$\mu_1 < \mu_2 \ldots < \mu_n < \ldots$$

is satisfied, every sequential term of the series (3.24), (3.25) will be negligibly smaller than the previous one as $\overline{x}$ grows. Therefore, beginning with a certain value of $\overline{x}_p^0$ it is reasonable to keep only the first terms of series expansions (3.24) and (3.25), that is, we obtain

$$\overline{C_p} = \overline{C_\infty} - F_1(\mu_1,\overline{y})\,e^{-\mathrm{Le}^*\mu_1^2\overline{x}_p}; \tag{4.1}$$

$$\overline{T_p} = \overline{T_\infty} + F_2(\mu_1,\overline{y})\,e^{-\mathrm{Le}^*\mu_1^2\overline{x}_p}, \tag{4.2}$$

where $\overline{C_\infty} = 1 - \overline{T_w}$; $\overline{T_\infty} = \overline{T_w}$ in accordance with the conditions (3.10);

$$F_1(\mu_1, \overline{y}) = 2\frac{(\cos\sqrt{\mathrm{Le}^*}\mu_1 - \overline{T}_w)\cos\mu_1\overline{y}}{\mu_1\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_1\sin\mu_1+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_1\cos\mu_1\right]};$$

$$F_2(\mu_1, \overline{y}) = 2\left\{\frac{-\overline{T}_w\cos\mu_1\cos\sqrt{\mathrm{Le}^*}\mu_1(1-\overline{y})}{\mu_1\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_1\sin\mu_1+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_1\cos\mu_1\right]}\right.$$

$$+\frac{\overline{T}_w\sqrt{\mathrm{Le}^*}\,K_a\sin\mu_1\sin\sqrt{\mathrm{Le}^*}\mu_1(1-\overline{y})}{\mu_1\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_1\sin\mu_1+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_1\cos\mu_1\right]}$$

$$\left.+\frac{\sqrt{\mathrm{Le}^*}\,K_a\sin\mu_1\sin\sqrt{\mathrm{Le}^*}\mu_1\overline{y}}{\mu_1\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_1\sin\mu_1+\sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_1\cos\mu_1\right]}\right\}.$$

Taking the logarithms of Eqs. (4.1),(4.2) we obtain

$$\ln(\overline{C}_\infty - \overline{C}_p) = \ln F_1(\mu_1, \overline{y}) - \mathrm{Le}^*\mu_1^2\,\overline{x}_p; \tag{4.3}$$

$$\ln(\overline{T}_p - \overline{T}_\infty) = \ln F_2(\mu_1, \overline{y}) - \mathrm{Le}^*\mu_1^2\,\overline{x}_p. \tag{4.4}$$

Thus the relationship between the logarithms of the excesses of concentration $\Delta\overline{C}_p = \overline{C}_\infty - \overline{C}_p$ and of temperature $\Delta\overline{T}_p = \overline{T}_p - \overline{T}_\infty$ and the longitudinal coordinate is a linear one. This fact permits us to carry out a full analogy between the heat and mass exchange at a given film section and the regular mode of nonstationary heat conduction problems.

The theory of the Kondratiev regular mode proceeds from the relation vspace*2mm

$$-\frac{\partial\ \ln\,\Delta T}{\partial t} = m, \tag{4.5}$$

where $m = \mathrm{const}$; $\Delta T = T_c - T$ is the temperature excess, T_c is the constant temperature of the ambient medium. Differentiating (4.3),(4.4) with respect to $\overline{x}_p$, we obtain

$$-\frac{\partial\ln\,\Delta\overline{T}_p}{\partial\,\overline{x}_p} = \mathrm{Le}^*\mu_1^2; \tag{4.6}$$

$$-\frac{\partial\ln\,\Delta\overline{C}_p}{\partial\,\overline{x}_p} = \mathrm{Le}^*\mu_1^2. \tag{4.7}$$

Expressions (4.6), (4.7) are completely similar to (4.5). The longitudinal coordinate $\overline{x}$ in them plays the role of time. The film section in which the heat and mass transfer proceeds in the mode of (4.6), (4.7) will be called the regular section. Let write Eqs. (4.6), (4.7) in the following form

$$-\frac{\partial\Delta\overline{T}_p}{\partial\,\overline{x}_p} = m\Delta\overline{T}_p, \quad -\frac{\partial\Delta\overline{C}_p}{\partial\overline{x}_p} = m\Delta\overline{C}_p \qquad (m = \mathrm{Le}^*\mu_1^2). \tag{4.8}$$

Expressions (4.8) look like boundary conditions of the third kind. Hence, the factor m plays the role of the dimensionless coefficients of heat and mass transfer. The peculiarity of the problem, namely, the proximity of the Le* values to unity (see Table 3), enables one to express the root values as functions of the basic criteria

Le^*, K_a. Since $Le^* \simeq 1$, let us represent it as $Le^* = 1 - (1 - Le^*) = 1 - \delta$ [$\delta = (1 - Le^*)$ is a small value]. In this case, the characteristic equation (3.21) can be written as

$$\tan\mu \tan\Big(1 - \frac{\delta}{2}\Big)\mu = \frac{1}{K_a(1 - \frac{\delta}{2})}. \tag{4.9}$$

At $\delta \to 0$, (4.7) degenerates into the equation $\tan^2\mu^{(0)} = 1/K_a$, which has roots defined by the expression

$$\mu^{(0)} = \arctan\Big(\pm\frac{1}{\sqrt{K_a}}\Big). \tag{4.10}$$

At small δ values it is natural to expect that the roots of Eq. (4.7) will be close to (4.10). Thus, let us expand the sought root in a power series in δ:

$$\mu = \mu^{(0)} + \delta\,\tilde{\mu}_1^{(0)} + \delta^2\,\tilde{\mu}_2^{(0)} + \cdots \tag{4.11}$$

Substituting this expression into the left-hand side of Eq. (4.9) and expanding tangents into Taylor series in the $\mu^{(0)}$ vicinity, we find

$$\tan\mu = \tan(\mu^{(0)} + \delta\tilde{\mu}_1^{(0)} + \cdots) = \tan\mu^{(0)} + \frac{1}{\cos^2\mu^{(0)}}\,\delta\tilde{\mu}_1^{(0)} + o(\delta^2),$$

$$\tan\Big[\Big(1 - \frac{\delta}{2}\Big)\mu\Big] = \tan\Big(\mu^{(0)} + \delta\tilde{\mu}_1^{(0)} - \frac{\delta}{2}\mu_0^{(0)} + \cdots\Big) = \tan\mu^{(0)} + \frac{1}{\cos^2\mu^{(0)}}\,\delta\Big(\tilde{\mu}_1^{(0)} - \frac{\mu^{(0)}}{2}\Big) + o(\delta^2),$$

$$\tan\mu\tan\Big(1 - \frac{\varepsilon}{2}\Big)\mu = \tan^2\mu^{(0)} + \delta\,\frac{\tan\mu^{(0)}}{\cos^2\mu^{(0)}}\Big(2\tilde{\mu}_1^{(0)} - \frac{\mu^{(0)}}{2}\Big) + o(\delta^2).$$

Expanding the right-hand side of Eq. (4.9) in a power series in δ up to the linear term, we obtain

$$\tan^2\mu^{(0)} + \frac{\tan\mu^{(0)}}{\cos^2\mu^{(0)}}\Big(2\tilde{\mu}_1^{(0)} - \frac{\mu^{(0)}}{2}\Big)\delta + o(\delta^2) = \frac{1}{K_a}\Big[1 + \frac{\delta}{2} + o(\delta^2)\Big].$$

For the zero- and first-order approximations we have, respectively,

$$\tan^2\mu^{(0)} = \frac{1}{K_a}, \qquad \frac{\tan\mu^{(0)}}{\cos^2\mu^{(0)}}\Big(2\tilde{\mu}_1^{(0)} - \frac{\mu^{(0)}}{2}\Big) = \frac{1}{2K_a},$$

whence

$$\tilde{\mu}_1^{(0)} = \frac{1}{4}\Big(\mu^{(0)} + \frac{\cos^2\mu^{(0)}}{K_a\tan\mu^{(0)}}\Big). \tag{4.12}$$

Making use of the obvious trigonometric identity $\cos^2\mu = 1/(1 + \tan^2\mu)$ we find finally from (4.12):

$$\tilde{\mu}_1^{(0)} = \frac{1}{4}\Big[\mu_0^{(0)} \pm \frac{1}{\sqrt{K_a}\,(1 + K_a)}\Big],$$

where the sign before the second addend is determined by the sign in (4.10).

Thus, the roots of (3.21) can be estimated approximately as

$$\mu = \mu^{(0)} + \frac{(1 - Le^*)}{4}\Big[\mu^{(0)} \pm \frac{1}{\sqrt{K_a}\,(1 + K_a)}\Big]. \tag{4.13}$$

Using the previously obtained concentration and temperature distributions in the film (4.1), (4.2) we find the local Sherwood and Nusselt numbers in the regular section:

$$Nu_p = \frac{\alpha_p x_p}{\lambda_L} = \frac{q_p x_p}{\lambda_L(T_e - T_0)} = 2\lambda_{\text{eff},L}\,Le^* K_a f(\mu_1)\overline{x}_p\,e^{-Le^*\mu_1^2\overline{x}_p}; \tag{4.14}$$

$$Sh_p = \frac{\beta_p x_p}{D_L} = \frac{j_p x_p}{\rho D_L(T_e - T_0)} = 2D_{\text{eff},L}\,Pe_Q^*\,\overline{x}_p f(\mu_1)\,e^{-Le^*\mu_1^2\overline{x}_p}, \tag{4.15}$$

where

$$f(\mu_1) = \frac{\sin\mu_1(\cos\sqrt{\mathrm{Le}^*}\mu_1 - \overline{T}_w)}{\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_1\sin\mu_1 + \sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_1\cos\mu_1\right]}.$$

Thus the obtained solutions enable one to determine completely the heat exchange in the regular section. However, for the practical applications of the obtained results it is necessary to know the coordinate $\overline{x}_p^0$ of the initial point of the film regular section.

For the regular section, as has been elucidated previously, it is sufficient to keep only the first terms of the series (3.24) and (3.25). Thus it is necessary to estimate the remainders of the series (3.24) and (3.25) at $\overline{y}=1$, that is, on the film surface. Using the characteristic equation (3.21), we transform the functions $F_1(\mu_n,1)$, $F_2(\mu_n,1)$ as follows:

$$F_1(\mu_n,1) = 2\frac{(\cos\sqrt{\mathrm{Le}^*}\mu_n - \overline{T}_w)\cos\mu_n}{\mu_n\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_n\sin\mu_n + \sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_n\cos\mu_n\right]}$$

$$= \frac{2}{\mu_n}\frac{\left(1-\dfrac{\overline{T}_w}{\cos\sqrt{\mathrm{Le}^*}\mu_n}\right)\cot\mu_n}{1+\mathrm{Le}^*K_a+\dfrac{1+K_a}{K_a}\cot^2\mu_n}. \tag{4.16}$$

Since the value of Le^* is close to unity, it is reasonable to assume that the transformed expression of the function $F_2(\mu_n,1)$ must coincide with that of the function $F_1(\mu_n,1)$. Let us show that this is the case indeed:

$$F_2(\mu_n,1) = \frac{2}{\mu_n}\frac{\sqrt{\mathrm{Le}^*}K_a\sin\mu_n\sin\sqrt{\mathrm{Le}^*}\mu_n - \overline{T}_w\cos\mu_n}{\left[(1+\mathrm{Le}^*K_a)\cos\sqrt{\mathrm{Le}^*}\mu_n\sin\mu_n + \sqrt{\mathrm{Le}^*}(K_a+1)\sin\sqrt{\mathrm{Le}^*}\mu_n\cos\mu_n\right]}$$

$$= \frac{2}{\mu_n}\frac{\sqrt{\mathrm{Le}^*}K_a\tan^2\sqrt{\mathrm{Le}^*}\mu_n - \dfrac{\overline{T}_w}{\cos\sqrt{\mathrm{Le}^*}\mu_n}\cot\mu_n}{1+\mathrm{Le}^*K_a+\dfrac{1+K_a}{K_a}\cot^2\mu_n}. \tag{4.17}$$

From expression (4.13) one can see that for values of Le^* close to unity terms of zero order are already a good approximation to the solution of the characteristic equation (3.21). Taking this into account, we obtain from (4.17):

$$F_2(\mu_n,1) = \frac{2}{\mu_n}\frac{1-\dfrac{\overline{T}_w}{\cos\sqrt{\mathrm{Le}^*}\mu_n}\cot\mu_n}{1+\mathrm{Le}^*K_a+\dfrac{1+K_a}{K_a}\cot^2\mu_n},$$

which identically coincides with (4.16) for the function $F_1(\mu_n,1)$.

Thus, at $\mathrm{Le}^*\simeq 1$ there is a complete mutual similarity of the concentration and temperature fields inside the film. Expression (4.16) can be simplified further. To this end let us utilize the relations following from the characteristic equation and from the condition $\mathrm{Le}^*\simeq 1$:

$$\cot\mu_n \simeq \pm\sqrt{K_a},\quad \cot^2\mu_n \simeq K_a,\quad \cos\sqrt{\mathrm{Le}^*}\,\mu_n \simeq \pm\sqrt{\frac{K_a}{1+K_a}}. \tag{4.18}$$

Hence we obtain finally

$$F(\mu_n,1) = F_1(\mu_n,1) = F_2(\mu_n,1) = \frac{1}{\mu_n}\frac{\pm\left(1\mp\sqrt{\dfrac{1+K_a}{K_a}}\overline{T}_w\right)\sqrt{K_a}}{1+K_a}. \tag{4.19}$$

Using (4.19) and the characteristic equation $\tan^2\mu_n = 1/K_a$, one can readily show that series (3.24) can be written as

$$\Delta\overline{C} = \overline{C_\infty} - \overline{C} = a_1(x) + a_2(x) - a_3(x) - a_4(x) + a_5(x) + a_6(x) - \dots, \tag{4.20}$$

where $a_n = |F(\mu_n, 1)|\, e^{-\text{Le}^*\mu_n^2\overline{x}} > 0$. It is obvious that the coefficients a_n monotonically tend to zero (so that $a_n > a_{n+1}$). For the regular section one can neglect all the terms of the series (4.20) except the first one. Let us estimate the error of such an approximation. In the regular section expression (4.20) becomes

$$\Delta\overline{C_p} - a_1(\overline{x}) = b_1(\overline{x}) - b_2(\overline{x}) + b_3(\overline{x}) - \dots \qquad (b_n(\overline{x}) = a_{n+1}(\overline{x}) - a_{n+2}(\overline{x}))$$

It is easy to show that the terms $b_n(\overline{x})$ for an arbitrary x are positive and constitute a monotonically decreasing sequence tending to zero. The series on the right-hand side satisfies all the conditions of the Leibniz test; therefore, one can write the estimate at once

$$\Delta\overline{C_p} - a_1(\overline{x}) < b_1(\overline{x}).$$

In the regular section the following inequality must be satisfied:

$$\Delta\overline{C_p} - a_1(\overline{x}) < \delta$$

($\delta > 0$ is an arbitrarily small value given beforehand).

Hence, the coordinate of the initial point of the regular section is a solution to the equation

$$b_1(\overline{x}_p^0) = a_2(\overline{x}_p^0) - a_3(\overline{x}_p^0) = |F(\mu_2, 1)|\, e^{-\text{Le}^*\mu_2^2\overline{x}_p^0} - |F(\mu_3, 1)|\, e^{-\text{Le}^*\mu_3^2\overline{x}_p^0} = \delta. \tag{4.21}$$

Since (4.21) is transcendental, one must perform numerical calculations to solve it. However the peculiarity of the problem is that the functions on the left-hand side of (4.21) have different scales. This makes it possible to find an approximate analytical expression for the root.

Using (4.18), (4.19) one can see for himself that $\mu_2|F(\mu_2, 1)| = \mu_3|F(\mu_3, 1)| = \gamma$. Thus Eq. (4.21) becomes

$$\frac{e^{-\text{Le}^*\mu_2^2\overline{x}_p^0}}{\mu_2} - \frac{e^{-\text{Le}^*\mu_3^2\overline{x}_p^0}}{\mu_3} = \frac{\delta}{\gamma}. \tag{4.22}$$

To further simplify the expressions, let us denote the root to be found by the letter z. To solve Eq. (4.22) we use the Newton–Raphson–Kantorovich method. It is significant that the root value of the equation is essentially determined by the first term in the left-hand side of Eq. (4.22). Thus, neglecting the second term, we can easily find a zero approximation for the root:

$$z_0 = \frac{1}{\text{Le}^*\mu_2^2} \ln \frac{\delta\mu_2}{\gamma}.$$

Consider the function

$$G(z) = \frac{e^{-\text{Le}^*\mu_2^2 z}}{\mu_2} - \frac{e^{-\text{Le}^*\mu_3^2 z}}{\mu_3} - \frac{\delta}{\gamma}.$$

Expanding $G(z)$ into a Taylor series in the vicinity of z_0 up to the linear term we obtain

$$G(z) = G(z_0) + \frac{dG(z_0)}{dz}(z - z_0). \tag{4.23}$$

Considering z as a root to be found, from (4.23) we get

$$z = z_0 - \frac{G(z_0)}{dG(z_0)/dz}.$$

Computing the values of the functions $G(z)$, $dG(z)/dz$ at z_0 point, we obtain finally the coordinate of the initial point of the regular section:

$$\overline{x}_p^0 = \frac{1}{\mathrm{Le}^*\mu_2^2}\ln\frac{\gamma}{\delta\mu_2} - \frac{1}{\mathrm{Le}^*\mu_3}\frac{\left(\frac{\delta\mu_2}{\gamma}\right)^{n^2}}{\mu_2^2\frac{\delta}{\gamma} - \mu_3\left(\frac{\delta\mu_2}{\gamma}\right)^{n^2}} \qquad (n = \mu_3/\mu_2).$$

5. Adiabatic Surface. The physical and mathematical formulation of the problem in nondimentional form (2.14)–(2.17) holds also for this case, but the boundary condition (2.18) must be replaced by (2.19). The Laplace transform technique can also be used to solve the equations. We write the general solution in the image space by analogy with the isothermal surface case:

$$\tilde{\tilde{C}} = \frac{1}{s}\frac{\sinh\sqrt{s}\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\,\overline{y}}{\sinh\sqrt{s}\cosh\sqrt{\frac{s}{\mathrm{Le}^*}} + \sqrt{\mathrm{Le}^*}\,K_a\sinh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh\sqrt{s}} = \frac{1}{s}\frac{\Phi_1(s)}{\varphi(s)}; \tag{5.1}$$

$$\tilde{\tilde{T}} = \frac{1}{s}\frac{\sqrt{\mathrm{Le}^*}\,K_a\sinh\sqrt{s}\cosh\sqrt{\frac{s}{\mathrm{Le}^*}}\,\overline{y}}{\sinh\sqrt{s}\cosh\sqrt{\frac{s}{\mathrm{Le}^*}} + \sqrt{\mathrm{Le}^*}\,K_a\sinh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh\sqrt{s}} = \frac{1}{s}\frac{\Phi_2(s)}{\varphi(s)}. \tag{5.2}$$

Before proceeding to perform the inverse transform of expressions (5.1) and (5.2), let us find asymptotes in the initial section of the film and at infinity.

Since at the initial section the thermal and diffusion layers are thin, and the solution does not "feel" the boundary conditions on the wall, the asymptotics of the expressions (5.1) and (5.2) for the case must coincide with the corresponding solutions (3.8) and (3.9) for the isothermal surface case.

One can easily check to be certain that this is really the case by using the scheme of reversing the solution in the image space which was offered in Section 3.

It is natural that the plate length-averaged Sherwood and Nusselt numbers coincide, respectively, with the values (3.13) and (3.14). Similarly we find the asymptotic values of temperature and concentration at infinity:

$$\overline{C}_\infty = \frac{1}{1+K_a}, \qquad \overline{T}_\infty = \frac{K_a}{1+K_a}.$$

Now we proceed to invert the Laplace transform of the solution (5.1) and (5.2). These expressions do not satisfy the conditions of the expansion theorem. However, it is easy to show that rewriting them as

$$\tilde{\tilde{C}} = \frac{\Phi_1(s)/\sqrt{s}}{s\varphi(s)/\sqrt{s}} = \frac{\Psi_1(s)}{s\varphi_0(s)}, \qquad \tilde{\tilde{T}} = \frac{\Phi_2(s)/\sqrt{s}}{s\varphi(s)/\sqrt{s}} = \frac{\Psi_2(s)}{s\varphi_0(s)}, \tag{5.3}$$

we get the quotients of entire transcendental functions. Hence, the expansion theorem can be applied to the expressions written in such a form. It is seen from (5.3) that poles of the functions $\tilde{\tilde{C}}\,(s)$ and $\tilde{\tilde{T}}\,(s)$ coincide. Besides the obvious zero root, there are a countable set of simple roots which are determined by the equation

$$\sinh\sqrt{s}\cosh\sqrt{\frac{s}{\mathrm{Le}^*}} + \sqrt{\mathrm{Le}^*}\,K_a\sinh\sqrt{\frac{s}{\mathrm{Le}^*}}\cosh\sqrt{s} = 0. \tag{5.4}$$

Note that the zero root is a single root since $\lim\limits_{s\to 0}\varphi_0(s) \neq 0$. Using relations (3.19) and introducing a new variable $\mu = i\sqrt{s/\text{Le}^*}$, we obtain from (5.4) the characteristic equation

$$\tan\sqrt{\text{Le}^*}\,\mu_n + \sqrt{\text{Le}^*}\,K_a \tan\mu_n = 0. \tag{5.5}$$

Since $\sqrt{\text{Le}^*}$ in the general case is an irrational number, no additional roots appear, and, consequently, the zero root and the roots of Eq. (5.5) are the only poles of functions (5.3). It is known from the operational calculus that if $\Phi(s)$ and $\varphi(s)$ are extended polynomials with respect to s, and $\Phi(s) = s^k\Phi_1(s)$, $\varphi(s) = s^k\varphi_1(s)$ ($|k| < 1$), then

$$\lim_{s\to s_n} \frac{\Phi(s)}{\varphi'(s)} = \lim_{s\to s_n} \frac{\Phi_1(s)}{\varphi_1'(s)}, \tag{5.6}$$

where s_n are the roots of the equation $\varphi_1(s) = 0$.

Using (5.6) and inverse transform formulas (3.22) we obtain

$$\overline{C} = \frac{1}{1+K_a} - 2\sum_{n=1}^{\infty} \frac{\sin\sqrt{\text{Le}^*}\,\mu_n \cos\mu_n\overline{y}}{\mu_n B(\mu_n)}\, e^{-\text{Le}^*\mu_n^2\overline{x}}; \tag{5.7}$$

$$\overline{T} = \frac{K_a}{1+K_a} - 2\sum_{n=1}^{\infty} \frac{\sqrt{\text{Le}^*}\,K_a \sin\mu_n \cos\sqrt{\text{Le}^*}\mu_n\overline{y}}{\mu_n B(\mu_n)}\, e^{-\text{Le}^*\mu_n^2\overline{x}}, \tag{5.8}$$

where $B(\mu_n) = (1+\text{Le}^*K_a)\sin\mu_n \sin\sqrt{\text{Le}^*}\mu_n - \sqrt{\text{Le}^*}(1+K_a)\cos\mu_n\cos\sqrt{\text{Le}^*}\mu_n$.

6. Analysis of the Solution. The characteristic equation (5.5) is a transcendental one, and in the general case of arbitrary Le* values it can be solved only by numerical methods. However, restricting our consideration to values of Le* close to unity, we can derive the asymptotic solution of Eq. (5.5), as well as for the isothermal surface case. Note that at Le* = 1 the roots can be easily found:

$$\mu^{(0)} = \arctan 0.$$

Therefore we shall search for the solution to Eq. (5.5) in the form of expansion (4.1) with respect to the small parameter $\delta = 1 - \text{Le}^*$. Omitting calculations similar to those in Section 4, we obtain

$$\mu_{2n} = \mu_n^{(0)}\left[1 + \frac{1-\text{Le}^*}{2(1+K_a)}\right]. \tag{6.1}$$

It is significant that (6.1) gives only the roots located in the range

$$\pi(n-1/2)\,/\sqrt{\text{Le}^*} < \mu_{2n} < (1/2+n)\pi, \quad n = 1, 2, \ldots$$

The roots μ_{2n+1} located in the range

$$(1/2+n)\pi < \mu_{2n+1} < (n+1/2)\pi\,/\sqrt{\text{Le}^*}, \quad n = 1, 2, \ldots$$

are already not close to the roots of the unperturbed equation $\tan\mu = 0$, hence we use a different approach to find them. Note that the length $l = (1/2+n)\pi(\delta/2)$ of the interval $(\pi(1/2+n),\ (n+1/2)\pi\,/\sqrt{\text{Le}^*})$ is small even at moderate n values. Therefore it is possible to use the asymptotic expansions of the functions $\tan\mu$ and $\tan\sqrt{\text{Le}^*}\mu$ close to their asymptotes $y = \pi(1/2+n)$ and $y = \pi(1/2+n)\,/\sqrt{\text{Le}^*}$, respectively:

$$\begin{aligned} \tan\sqrt{\text{Le}^*}\mu &\simeq \frac{1}{(1/2+n)\pi - \sqrt{\text{Le}^*}\mu} && \text{at} \quad \mu \to (1/2+n)\pi\,/\sqrt{\text{Le}^*} - 0, \\ \tan\mu &\simeq \frac{1}{(1/2+n)\pi - \mu} && \text{at} \quad \mu \to (1/2+n)\pi + 0. \end{aligned} \tag{6.2}$$

Substitution of expansions (6.2) into the characteristic equation (5.5) yields

$$\mu_{2n+1} = \frac{(2n+1)\pi}{2}\,\frac{1+\sqrt{\mathrm{Le}^*}K_a}{1+\mathrm{Le}^*K_a}, \quad n = 0, 1, \ldots \tag{6.3}$$

Since the sequence of roots (6.1), (6.3) is monotonically increasing, there is a section of the film in which the regular mode of heat and mass transfer is realized as in the case of an isothermal surface. Consequently, one can write ($\Delta\overline{C}_p = \overline{C}_p - \overline{C}_\infty$, $\Delta\overline{T}_p = \overline{T}_\infty - \overline{T}_p$):

$$\ln\Delta\overline{C}_p = \ln\, F_1(\mu_1,\overline{y}) - \mathrm{Le}^*\mu_1^2\overline{x}_p; \tag{6.4}$$

$$\ln\Delta\overline{T}_p = \ln\, F_2(\mu_1,\overline{y}) - \mathrm{Le}^*\mu_1^2\overline{x}_p, \tag{6.5}$$

where

$$F_1(\mu_1,\overline{y}) = 2\,\frac{\sin\sqrt{\mathrm{Le}^*}\mu_1\cos\mu_1\overline{y}}{\mu_1 B(\mu_1)};$$

$$F_2(\mu_1,\overline{y}) = 2\,\frac{\sqrt{\mathrm{Le}^*}K_a\sin\mu_1\cos\sqrt{\mathrm{Le}^*}\mu_1\overline{y}}{\mu_1 B(\mu_1)}.$$

Differentiating expressions (6.4), (6.5) with respect to $\overline{x}_p$ and making use of (6.3), we finally obtain

$$-\frac{\partial\Delta\overline{T}_p}{\partial\overline{x}_p} = m\Delta\overline{T}_p, \qquad -\frac{\partial\Delta\overline{C}_p}{\partial\overline{x}_p} = m\Delta\overline{C}_p,$$

$$m = \frac{\pi^2}{4}\left(\frac{\sqrt{\mathrm{Le}^*}+\mathrm{Le}^*K_a}{1+\mathrm{Le}^*K_a}\right)^2 \simeq \frac{\pi^2}{4}\,\frac{1+K_a}{1+\mathrm{Le}^*K_a}\,\mathrm{Le}^*.$$

$$-\frac{\partial\Delta\overline{T}_p}{\partial\overline{x}_p} = m\Delta\overline{T}_p, \qquad -\frac{\partial\Delta\overline{C}_p}{\partial\overline{x}_p} = m\Delta\overline{C}_p, \qquad m = \frac{\pi^2}{4}\left(\frac{\sqrt{\mathrm{Le}^*}+\mathrm{Le}^*K_a}{1+\mathrm{Le}^*K_a}\right)^2 \simeq \frac{\pi^2}{4}\,\frac{1+K_a}{1+\mathrm{Le}^*K_a}\,\mathrm{Le}^*.$$

The expressions for the local Nusselt and Sherwood numbers in the regular section of the film practically coincide with (4.14) and (4.15), except the multiplier

$$f(\mu_1) = \frac{\sin\mu_1\sin\sqrt{\mathrm{Le}^*}\mu_1}{B(\mu_1)}.$$

To determine the initial coordinate of the regular section $\overline{x}_p^0$ we rewrite the functions $F_1(\mu_n, 1)$, $F_2(\mu_n, 1)$ as

$$|F_1(\mu_n,1)| = |F_2(\mu_n,1)| = 2\,\frac{K_a|\tan\mu_n|}{\mu_n[K_a(1+\mathrm{Le}^*K_a)\tan^2\mu_n + (1+K_a)]}. \tag{6.6}$$

Substituting the Taylor series expansion of the function $\tan\sqrt{\mathrm{Le}^*}\mu$ in the vicinities of the "even" roots μ_{2n} into the characteristic equation (5.5) we get

$$\tan^2\mu_{2n} - \frac{2p}{\delta\mu_{2n}}\tan\mu_{2n} + 1 = 0, \tag{6.7}$$

where $\delta = 1 - \mathrm{Le}^*$; $p = 1 + [(1+\mathrm{Le}^*)2]\,K_a$.

Equation (6.7) permits us to express the value of $\tan\mu_{2n}$ in terms of the value of the corresponding root μ_{2n}

$$\tan\mu_{2n} \simeq \frac{\delta\mu_{2n}}{2p}. \tag{6.8}$$

Using (6.1) and (6.8), we find from (6.6)

$$|F_1(\mu_{2n},1)| = |F_2(\mu_{2n},1)| = \frac{1}{An^2+B},$$

where

$$A = \frac{\pi^2}{4}\frac{(1+\mathrm{Le}^* K_a)\delta}{p}; \qquad B = \frac{(1+K_a)p}{K_a\delta}.$$

Let us define the form of the functions $|F_1(\mu, 1)|$, $|F_2(\mu, 1)|$ for roots with odd subscripts, that is, for $\mu = \mu_{2n+1}$. Substitution of expressions (6.2) and (6.3) into (6.6) yields

$$|F_1(\mu_{2n+1}, 1)| = |F_2(\mu_{2n+1}, 1)| = \frac{1}{(2n+1)^2 C + D},$$

where

$$C = \frac{\pi^2\delta(1+K_a)(1+\sqrt{\mathrm{Le}^*}K_a)}{16(1+\mathrm{Le}^* K_a)^2}; \qquad D = \frac{(1+\mathrm{Le}^* K_a)(1+\sqrt{\mathrm{Le}^*}K_a)}{\delta K_a}.$$

Using (6.2) and (6.8), it is easy to show that series (5.7) and, consequently, series (5.8) are alternating series and satisfy all the conditions of the Leibniz test. Thus the coordinate of the initial point $\overline{x}_p^0$ of the regular section is to be determined from the inequality

$$\frac{1}{A+B}\mathrm{e}^{-\mathrm{Le}^*\mu_2^2\overline{x}_p^0} \leqslant \varepsilon.$$

Solving it with respect to $\overline{x}_p^0$ and substituting the expression (6.1) for μ_2, we obtain

$$\overline{x}_p^0 \geqslant -\frac{\ln[(A+B)\varepsilon]}{\pi^2\mathrm{Le}^*\left[1+\frac{2(1-\mathrm{Le}^*)}{(1+K_a)}\right]}.$$

In conclusion it may be said that the results obtained in the present work prove the existence of a range of bed ball diameters for which the heat and mass transfer rate increases considerably (2–4 times) as compared to the case of film absorption on a smooth plate. The general analytical solutions are found both for isothermal and for adiabatic surfaces, from which the known boundary layer approximation solutions of the nonisothermal film absorption problem follow. It is proved that when the bed ball diameter is greater than 1.5 mm, a full analogy is observed between heat and mass transfer processes. This feature is connected with the proximity of the effective Lewis number to unity. It has been shown that beginning at a certain distance from the inlet of the film the regular regime of heat and mass exchange is realized. A linear dependence of the logarithms of the temperature and concentration excesses on the longitudinal coordinate is characteristic for this mode. For both types of boundary conditions explicit analytical expressions are obtained for the coordinates of the initial point of the regular section.

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